Indian Statistical Institute
B.Math.(Hons.) II Year
First Semester Back Paper Exam, 2006-07
Algebra III

Time: 3 hrs | Date: -01-07 | Total Marks : 50

Instructor: J Biswas

Attempt all questions.

1. Prove that the ideal generated by the polynomials x^2+y^2-1, x^2-y+1 and $xy-1$ is the whole ring $\mathbb{C}[x,y]$.
2. Is the ring $\mathbb{F}_3[x]/(x^3+x+1)$ a field? Justify your answer.
3. Let R be the ring $\mathbb{Z}[\sqrt{3}]$. Prove that a prime integer p is a prime element of R if and only if the polynomial x^2-3 is irreducible in $\mathbb{F}_p[x]$.
4. Let p be a prime integer. Prove that $x^{p-1}+x^{p-2}+\ldots+x^2+x+1$ is irreducible in $\mathbb{Q}[x]$.
5. Let I be an ideal of a ring (commutative with identity) R. If R/I is a free R-module then show that $I=0$.
6. Determine the number of isomorphism clases of abelian groups of order 400.
7. Reduce the matrix $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ to diagonal form by integer row and column operations. Let $V=\mathbb{Z}^2$ and let $L=AV$. Draw the sublattice L and find commensurable bases of V and L.
8. Determine the Jordan form of the matrix $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$.